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ON DISTURBANCE GROWTH MECHANISMS IN A BUOYANCY INDUCED FLOW

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NOMENCLATURE

G^*, $5[Gr_x^*/5]^{1/5}$;
G_N^*, value of G^* at the neutral condition;
G_{NL}^*, Value of G^* at which non-linear disturbance mechanisms arise;
Gr_x^*, $\dfrac{gx^3}{\nu^2}\dfrac{q''x}{k}$;
G_{Tr}^*, value of G^* at which transition begins;
q'', surface heat flux;
u', periodic part of longitudinal velocity disturbance;
U, characteristic local velocity;
x, streamwise coordinate.

Greek symbols
β, generalized disturbance frequency;
ν, kinematic viscosity.

1. INTRODUCTION

THE ACCUMULATION of specific information concerning disturbance growth and flow transition mechanisms of vertical natural convection flows, leads to detailed consideration of the interaction of naturally occurring background disturbances. Initial instability and downstream amplification of controlled disturbances are well understood. The predictions of linear theory are in very good agreement with experiments. See for example, Gebhart [1] and Gebhart and Mahajan [2]. Nonlinear growth characteristics have also been studied. Audunson and Gebhart [3] have calculated the resulting secondary mean motions. Jaluria and Gebhart [4] have measured such motions and changing disturbance characteristics with nonlinear effects.

On the other hand, there have been many observations of the subsequent event of transition, resulting from naturally occurring disturbances. See, e.g. Godaux and Gebhart [5], Jaluria and Gebhart [6] and Mahajan and Gebhart [7]. Here we consider the relation between what is known about growth of controlled disturbances and the way naturally occurring disturbances cause transition. We investigate this largely unknown aspect of natural convection transition by comparing growth in controlled experiments with abundant data of observed transition arising from naturally occurring disturbances.

We will estimate the effect the initial amplitude of an input disturbance has on progression toward transition, and on events during transition. Also considered is the effect of the frequency spectrum of the input disturbance on the disturbance frequency observed ahead of and during transition. The considerations which arise are discussed in the light of experimental data concerning disturbance magnitude.

2. EXPERIMENTAL RESULTS

Characteristics of naturally occurring disturbances were measured in a flow generated adjacent to a flat vertical surface, dissipating a uniform surface heat flux, in water. The arrangement and techniques are the same as those used by Jaluria and Gebhart [6] in investigating the events during transition, in a flow subject only to naturally occurring disturbances. Present observations clarify the mechanisms whereby disturbances initiate transition.

1. *Disturbance frequency and the beginning of transition*

A disturbance entering the boundary layer is characterized by its energy spectrum. When the magnitude is small, we may think of it as a group of individual periodic disturbances of different frequency and amplitude. In controlled experimentation, a disturbance of single frequency is studied. Such experiments, along with theory, have shown that disturbances in a very narrow band of frequency are much more rapidly amplified downstream.

A naturally occurring disturbance contributes a spectrum of frequencies, as confirmed by our hot-wire measurements of background disturbances. We have found no dominant frequency. However, selective amplification, or filtering, still occurs downstream, see [2]. Jaluria and Gebhart [4] found that this frequency persists even into and through the non-linear range. Essentially all the disturbance energy is concentrated there until the beginning of transition.

Even during transition the disturbance associated with the remaining laminar portion of the fluid remains at the theory-predicted filtered frequency. A different and higher dominant frequency is associated with the locally turbulent portion of the flow. This second frequency increases during transition, in nondimensional terms. This is discussed in detail in [6]. These frequency characteristics do not follow from the form of the initial naturally-occurring input disturbance. The observed frequencies in the non-linear range, and into transition, are determined only by the heat flux.

The transition Grashof number G_{Tr}^* denotes the location of first turbulence in the flow. See Jaluria and Gebhart [6]. Now, G_{Tr}^* was found to be a function of the heat flux input q''. Their measurements, and others, led Jaluria and Gebhart [6] to formulate the beginning of transition as:

$$G_{Tr}^*\left(\frac{\nu^2}{gx^3}\right)^{2/15} = E. \tag{1}$$

2. *Disturbance growth*

The initial finding of Godaux and Gebhart [5], that the Grashof number at which transition begins is a function of the heat flux q'', was refined and corroborated with new data, and with the results of others, by Jaluria and Gebhart

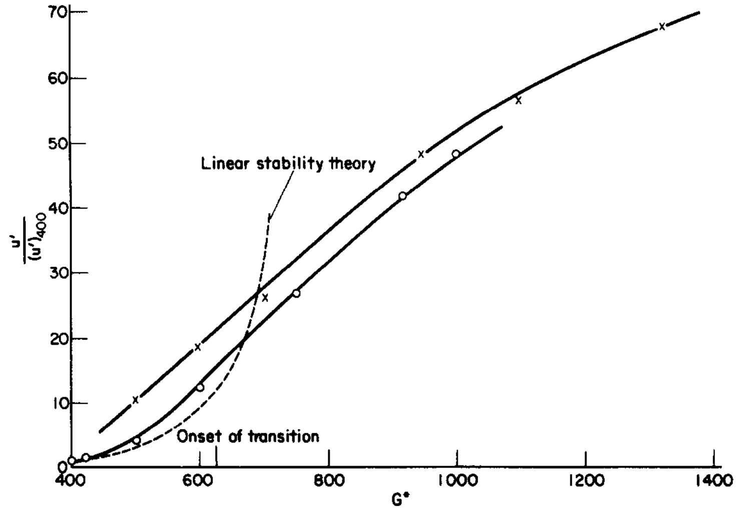

FIG. 1. Downstream amplification of the longitudinal velocity disturbance u' component, in terms of the disturbance amplitude at $G^* = 400$, $(u')_{400}$. Data: ○, $q'' = 210.5\,W/m^2$; ×, $q'' = 627.0\,W/m^2$.

[6]. The consequence, that a single value of E apparently signals transition in water, is a curious result, in several ways. It asserts that transition is independent of the magnitude of the random, naturally occurring background disturbances. One might reasonably surmise, on the contrary, that initially larger disturbances would reach and complete transition sooner, in x, for a given q''. However, if that is not the case, as suggested by the success of E, then different amounts of linear and nonlinear amplification must add up to about the same final disturbance amplitude at the same downstream location. We note, however, that there must, reasonably, be some appropriate limiting condition on the differences between linear and nonlinear amplification rates. A flow may not actually reach transition earlier for smaller initial disturbances. We will qualitatively investigate these questions.

In the linear growth range the amplification rate is independent of amplitude. At any given frequency, the ratio of the amplitude, A_2/A_1, after and before any particular downstream interval x_1 to x_2, depends only on G_1^* and G_2^*. However, the deviation of the growth rate from linear surely depends on the relative disturbance level u'/U, where u' is the amplitude of the longitudinal component of the local velocity disturbance and U is the maximum local mean flow velocity. This is suggested by linear analysis, nondimensionalization is accomplished in terms of this parameter. Measurements, by Jaluria and Gebhart [9], also indicate that significant nonlinear effects arise when this quantity attains a definite value, dependent on the heat flux level. Analogous results have been obtained in forced flow. See for example, Klebanoff *et al.* [10].

Measurements of subsequent nonlinear downstream growth are shown in Fig. 1 for naturally occurring disturbances and for q'' of 210.5 and 627.0 W/m^2. The measured maximum disturbance amplitudes u' are normalized by the value at $G^* = 400$, for $q'' = 210.5\,W/m^2$. For the lower q'', the growth rate becomes greater than linear at around $G^* = 440$. However, the rate soon decreases rapidly, and the actual amplitude becomes less than that by linear processes alone above about $G^* = 650$. Actual transition began at about $G^* = 625$ and was complete at around $G^* = 2000$. The transition limits for $q'' = 627.0\,W/m^2$ are $G^* = 500$ and 1400. The curves at the two values of q'', and at other values as well, are very similar. The growth at the higher q'' has deviated from linear at a lower value of G^*, indicating that higher disturbance levels were reached, at a given G^*, for higher q''.

We will use the measured nonlinear disturbance growth, to determine the effect of varying initial input disturbance amplitude on the beginning of transition, and on observed mechanisms during transition. The calculations of Audunson and Gebhart [3] were of secondary mean flows, not of nonlinear growth rates of u'.

Now a smaller input disturbance would have encountered nonlinear effects further downstream, where the energy level of the flow is greater. The flow kinetic energy flux varies as $G^{*5/2}$ and the total convected thermal energy, Q, as G^{*5}, for a given value of q''. Thus, an initially smaller disturbance may be expected to grow more rapidly after the appearance of nonlinear mechanisms. A detailed investigation of nonlinear disturbance growth might establish this proposition in general. However, we shall use the measured growth pattern for all postulated initial disturbance magnitudes, in the supposition that all disturbances follow it from initial nonlinear deviation.

We do not know which specific disturbance characteristic determines nonlinear events. The level u'/U determines their first appearance, as a function of the heat flux. Since the beginning of transition cannot be predicted by linear analysis, it is not known that the disturbance level u'/U determines either the onset of transition or the events subsequent thereto. On the other hand, the absolute disturbance magnitude may be the determining factor. We shall consider both these possibilities.

Consider $q'' = 210.5\,W/m^2$. Growth was linear until $G^* = 440$. For a smaller input disturbance, nonlinearity would be reached further downstream. However, U also increases downstream and the absolute disturbance amplitude would have to be higher if nonlinear deviation required the same disturbance level u'/U. For example, had the input disturbance amplitude been one-third that of the one that led to the data of Fig. 1, we calculate that nonlinear effects would first appear at $G^* = 535$. However, the actual physical amplitude of the disturbance would be, calculated from linear theory, about 15.7% higher than that measured at $G^* = 440$. The corresponding values for a disturbance of one-tenth initial magnitude are $G^* = 620$ and 29.4%. For a disturbance of three times the initial magnitude, nonlinear effects first appear at $G^* = 360$ where the physical amplitude is 14% lower than that measured.

Considering now nonlinear growth, we have plotted in Fig. 2 the measured nonlinear growth, at $q'' = 210.5\,W/m^2$, of the disturbance beyond the point of deviation, $G_{NL}^* = 440$, normalized by its value at G_{NL}^*. All disturbances, of varying

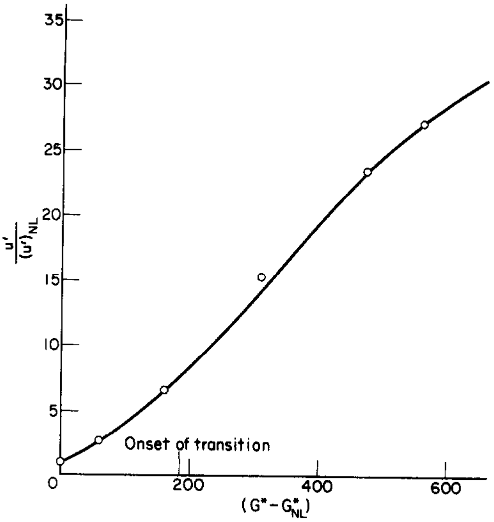

FIG. 2. Downstream amplification of u' after the first appearance of non-linear effects, G^*_{NL}, at $q'' = 210.5\ W/cm^2$, in terms of the disturbance amplitude at G^*_{NL}, $(u')_{NL}$.

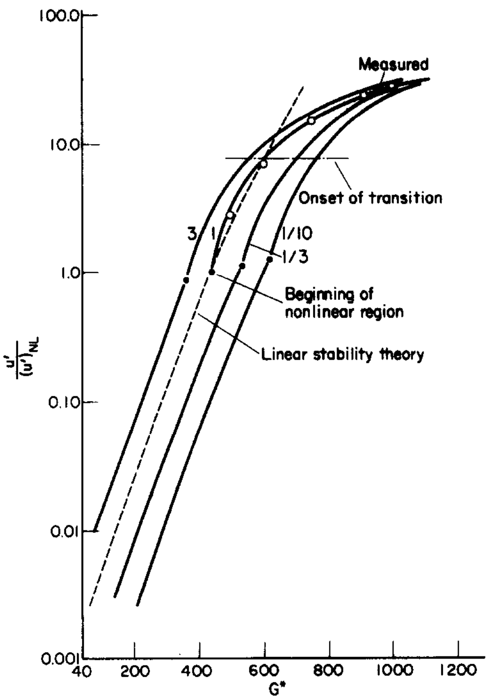

FIG. 3. Linear and nonlinear disturbance amplitude growth, for varying input disturbance amplitude. The values indicated refer to the ratio of the amplitude of the input background disturbance for each given curve, to that for the measured curve, at $q'' = 210.5\ W/cm^2$.

input amplitude, are assumed to grow according to theory in the linear range and according to Fig. 2, thereafter. The strong similarity between the two curves in Fig. 1, as well as other measurements, strongly supports this assumption.

We will investigate two suppositions; that the nonlinear mechanisms are determined, first, by the physical magnitude of the disturbance and, second, by its level relative to the mean flow. Since linear theory does not predict transition, the first may be more reliable.

We see from Fig. 2 that the disturbance magnitude u' at transition is 7.75 times that at the onset of nonlinear effects. Recall that the amplitude at nonlinear deviation for an initial amplitude of one-third, one-tenth and three times the experimental one is 15.7 and 29.4% higher and 14% lower, respectively. Therefore, for the first, the same disturbance amplitude is reached when the disturbance value on Fig. 2 is $7.75/1.157 = 6.7$. This is at $G^*_{Tr} = 695$, or a delay of about 11.2% in from the experimental value of $G^*_{Tr} = 625$. However, this is only about 5.4% in E, equation (1). The values for the one-tenth initial amplitude are $G^*_{Tr} = 768$, a delay of only 22.9% in G^* and 11.1% in E. For three times they are $G^*_{Tr} = 572$, which is 8.5% upstream in G^*. Clearly, the adjustment in growth sequence results in small changes in both G^*_{Tr} and E, for very different initial amplitudes.

In fact, nonlinear growth mechanisms operated in the laminar flow from $G^* = 440$ to the end of transition, at $G^* = 2000$. At an intermediate point during transition, at $G^* = 1090$, the growth has been by a factor of 30, accounting for the linear range as well, we find the remarkable result that the one-third, one-tenth and the three-fold initial amplitude disturbances would reach amplitudes, at $G^* = 1090$, which are within a few percent of the same value. Thus, the effect of initial amplitude on subsequent transition events is also very greatly reduced, due again to the flattening of the nonlinear growth curve.

These results are perhaps clearer in Fig. 3. Both the linear and non-linear regions are shown. The convergence to about the same amplitude, after the appearance of nonlinear effects, is very fast. We note, incidentally, that the nonlinear disturbance growth rate approaches a constant value during transition, as indicated by the linear form of the growth curves in Fig. 1.

The differences estimated for the onset of transition would be even further reduced either if smaller initial disturbances eventually grew more rapidly by nonlinear mechanisms, or if larger disturbances grew more slowly, due to the lower mean flow energy available for growth. Kinetic energy flux varies as $G^{*5/2} \propto E$, a quantity which correlates the onset of transition. The local kinetic energy flux at first nonlinear deviation, for a one-third initial magnitude, is about 63% higher than in the experimental circumstance. The possible effect of kinetic energy flux level on nonlinear disturbance growth merits further investigation. An interesting parallel question is the kinds of conditions on the amplification processes which ensure that weaker initial disturbances do not, in fact, lead to earlier transition.

Consider now the second possibility, wherein transition is characterized not by the physical magnitude, but by the relative disturbance level, i.e. by u'/U. The value of this quantity observed at the beginning of actual transition at $G = 625$, is at $G = 705$, 785 and 555 for one-third, one-tenth and three-fold amplitude disturbances, respectively. These values are 12.8% later, 25.6% later and 11.2% earlier, in G^*, and 6.3% larger, 12.1% larger and 5.4% smaller, in E. Again, we have calculated, on the basis of u'/U, the disturbance levels at $G = 1090$. They are again within a few percent of the measured level.

Thus, the two suppositions produce similar results. The beginning of transition is not greatly affected by a variation in the magnitude of the initial disturbance, nor are subsequent events during transition. Similar results were found using other measured growth rates. Our results lend support to a single transition parameter like E, which is independent of input disturbance amplitude.

During our experiments quoted there was very little change in the measured natural disturbance level. Repeatability was also very good. However, in similar experiments in a different experimental arrangement [9], presumably with a different background disturbance level, G^*_{Tr} was about 4% different.

In a similar manner, we may estimate the initial magnitude of the actual input disturbance, at the neutral stability location upstream, from linear theory predictions. For the measured frequency, this location is $G^*_N = 64$. From the magnitude at $G^* = 400$ the velocity disturbance at G^*_N

is calculated to be 3.6×10^{-5} cm/s, from the results of Hieber and Gebhart [8]. Since U at G_N^* is 0.2 cm/s, this disturbance level is only 0.018%. The disturbance level increases to 1.8% at $G^* = 440$, a 100-fold growth. The physical magnitude increases by about 450 fold.

We note that the velocity difference between two locations across the boundary layer around the inflexion point, and only 10^{-4} cm apart, is 2.4×10^{-5} cm/s at G_N^*. This places in proper perspective the very small magnitude of the actual physical disturbance, or mixing, which will induce nonlinear effects at $G^* = 440$ and cause transition at $G^* = 625$. A disturbance in the heat flux q'' of the order of 1% gives rise to the same disturbance magnitude. For even a much smaller disturbance, say one-tenth, transition will still correlate approximately to the same value of E. This suggests an experiment to determine G_{Tr}^* in a very quiet ambient with artificially induced disturbances, at various amplitudes.

3. CONCLUSIONS

For a naturally occurring input disturbance with no dominant components, the disturbance frequencies in actual transition are determined by frequency filtering. Variations in the input amplitude are estimated to have a small effect on the appearance of transition and on its progression. The initial input disturbance for observed transition is estimated and found to be very small. Flow interaction with background disturbances is very complex and future research is necessary to answer several remaining fundamental questions.

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GENERALIZED DIRECT EXCHANGE FACTORS FOR ISOTHERMAL MOLECULAR GASES

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NOMENCLATURE

A, dimensionless total band absorption;
A_s, dimensionless slab band absorption;
d, diameter;
D, band width parameter;
E_n, exponential integral;
E_ν, Planck radiosity;
F, defined by equation (14);
Gs, gas–surface exchange factor [dimensionless];
h, slab thickness;
K_n, nongray transfer functions;
L, length;
L_e, mean beam length;
L_0, optically thin mean beam length;
L_∞, optically thick mean beam length;
q_e^r, emitted radiative flux;
q_a^r, absorbed radiative flux;
R, radius;
R_ν, spectral boundary radiosity;
S, integrated band intensity;
Sg, surface–gas exchange factor;
t, path length;
T_g, gas temperature;
u, dimensionless mean beam length.

Greek symbols
β, band fine-structure parameter;
μ, direction cosine;
ρ_a, absorbing gas density;
τ_x, optical depth based upon length x;
ϕ, azimuthal angle.

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INTRODUCTION

ENGINEERING approximations for the analysis of radiative energy transfer from gases frequently involves the assumption of a uniform temperature. This proves to be a useful concept, for example, in a highly turbulent, well-stirred reactor and thus finds considerable application in the design of combustion devices for varied purposes. The isothermal assumption reduces the calculation of radiative transfer to the evaluation of transfer integrals depending only upon the geometry of the enclosure, with the radiation properties appearing parametrically. In general, closed form solutions are possible only for a limited number of simple configurations or in the limits of small and large optical paths. Hottel and Sarofim [1] have given a rather complete discussion of exchange areas and mean beam lengths for various geometries. Their discussion is largely limited to gray gases. These topics are also discussed from an occasionally different point of view by Siegel and Howell [2].